A nonparametric model for sensors used in a dynamic context

Nicolas Fischer, Eric Georgin, Amine Ismail, Emmanuel Vazquez, Laurent Le Brusquet

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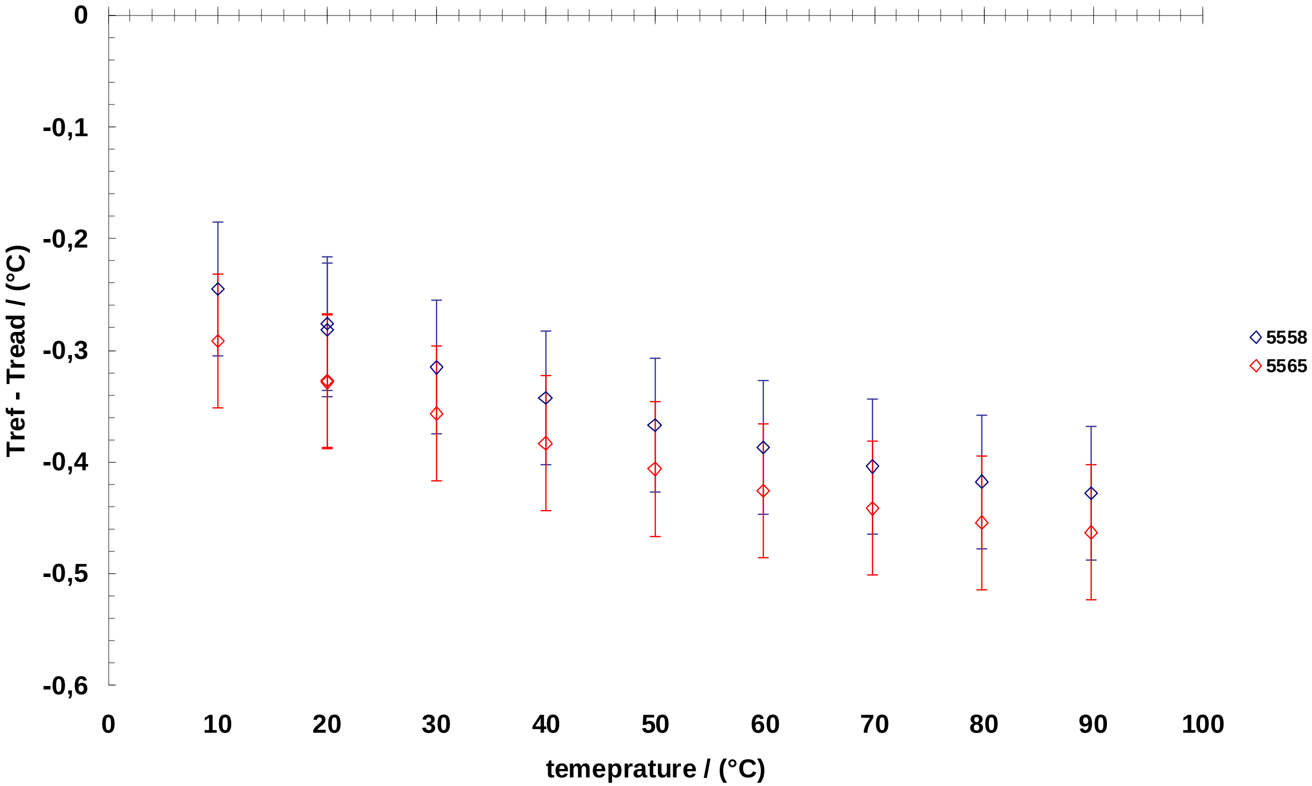
MEASUREMENT & STANDARDS
Keys to COMPETITIVENESS and A SAFER WORLD

Laboratoire national de métrologie et d’essais

Steady state or static conditions

Steady state

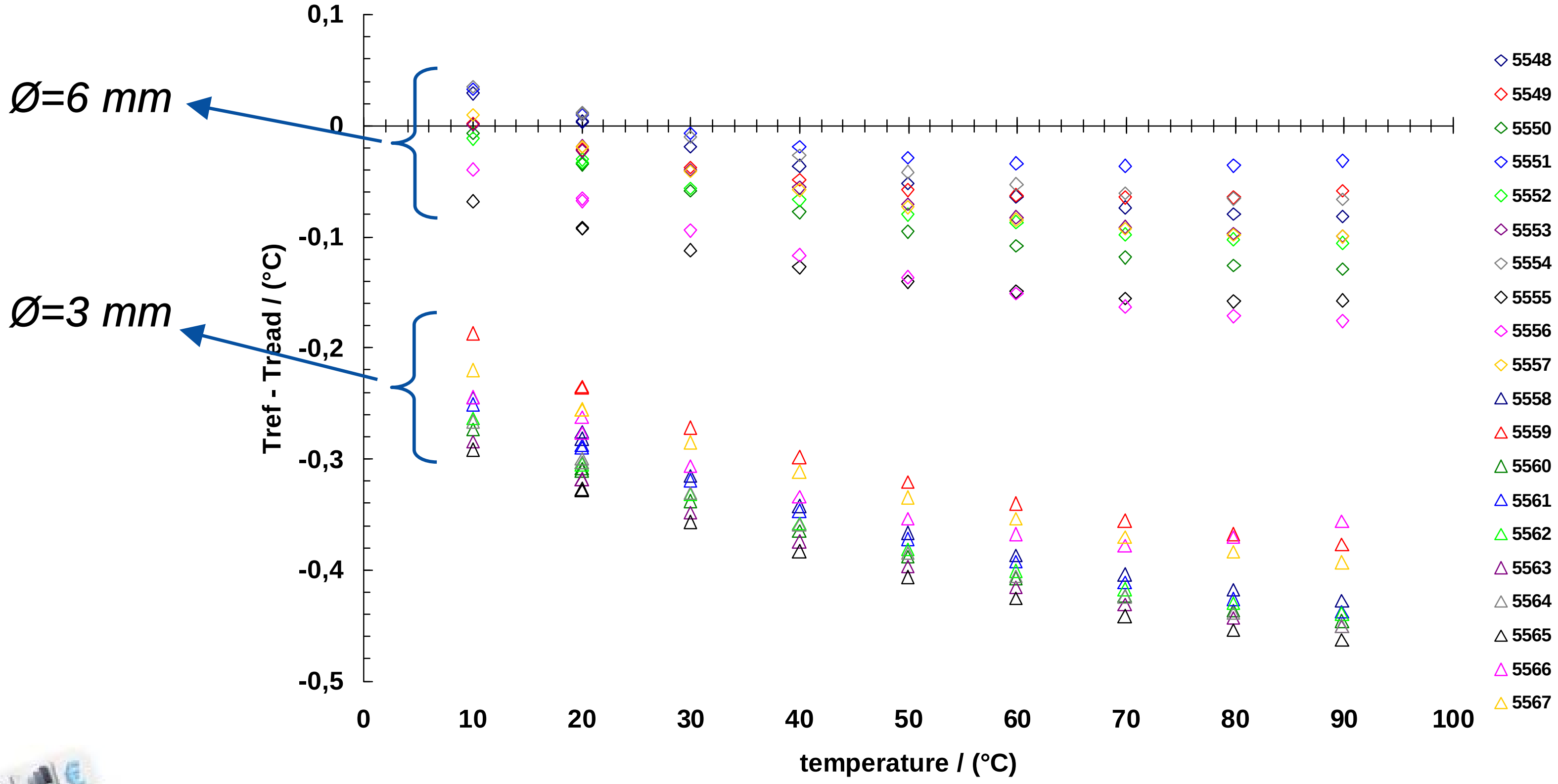
- Calibration procedure and uncertainty budget: well defined
- Example: calibration of 2 temperature sensors (Pt100 type)



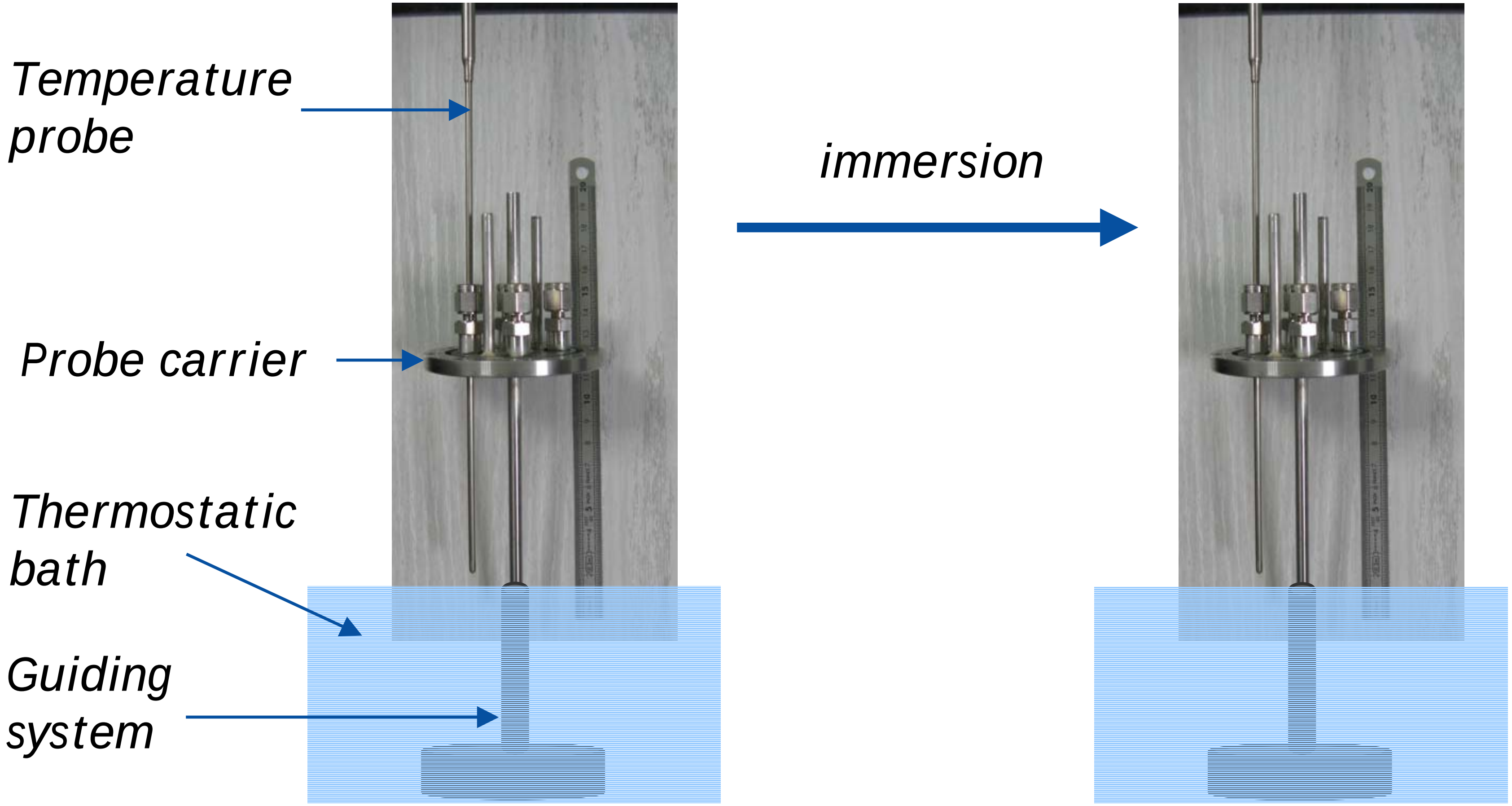
Dispersion of sensors in static conditions

Steady state

- 2 sets of Pt100 probes:
  - Ø= 6 mm and Ø= 3 mm
  - Calibrated between +10°C and 90°C



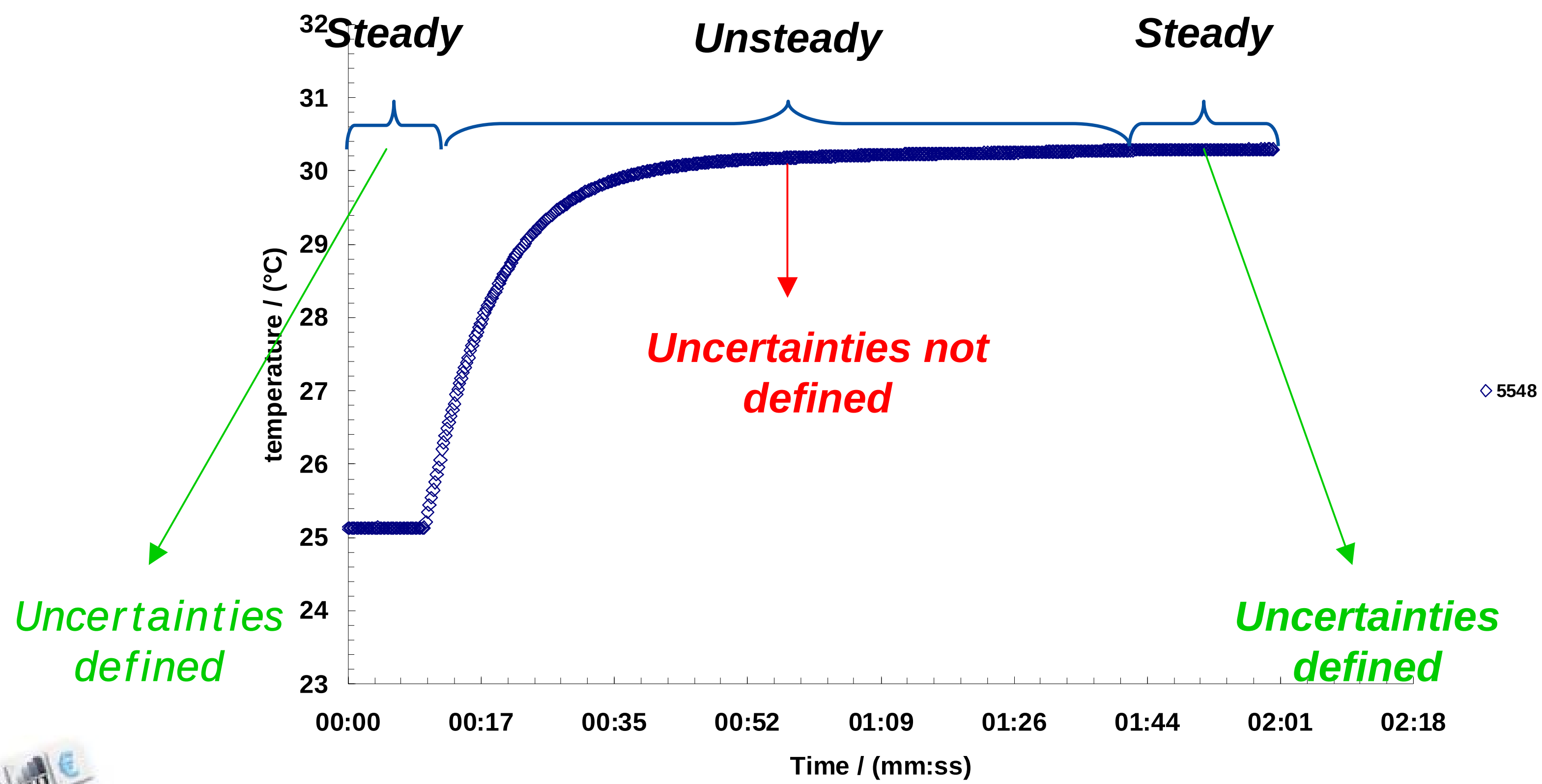
Experimental setup



Sensor response to a step excitation

Unsteady state

- Real measurement are often unsteady
- Example: **step excitation** in temperature, **1 probe**



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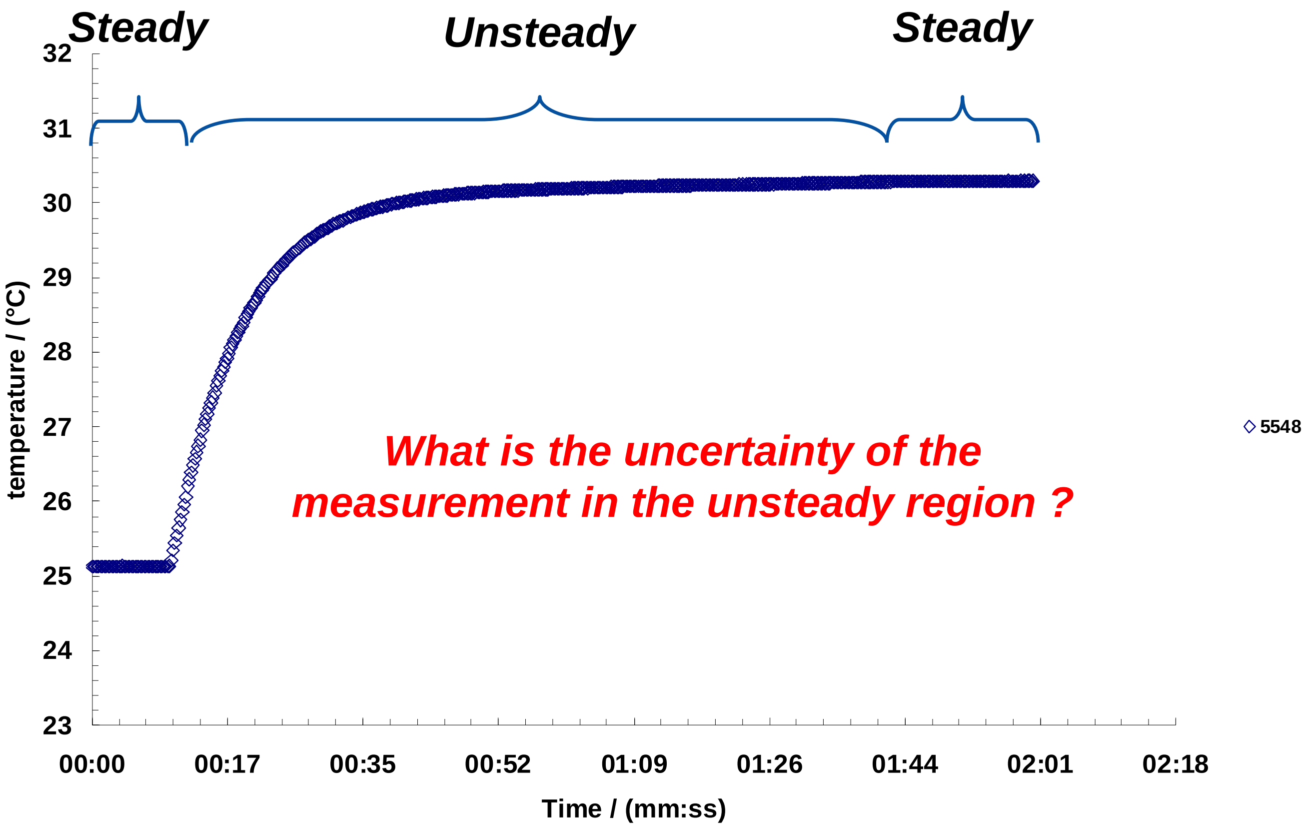
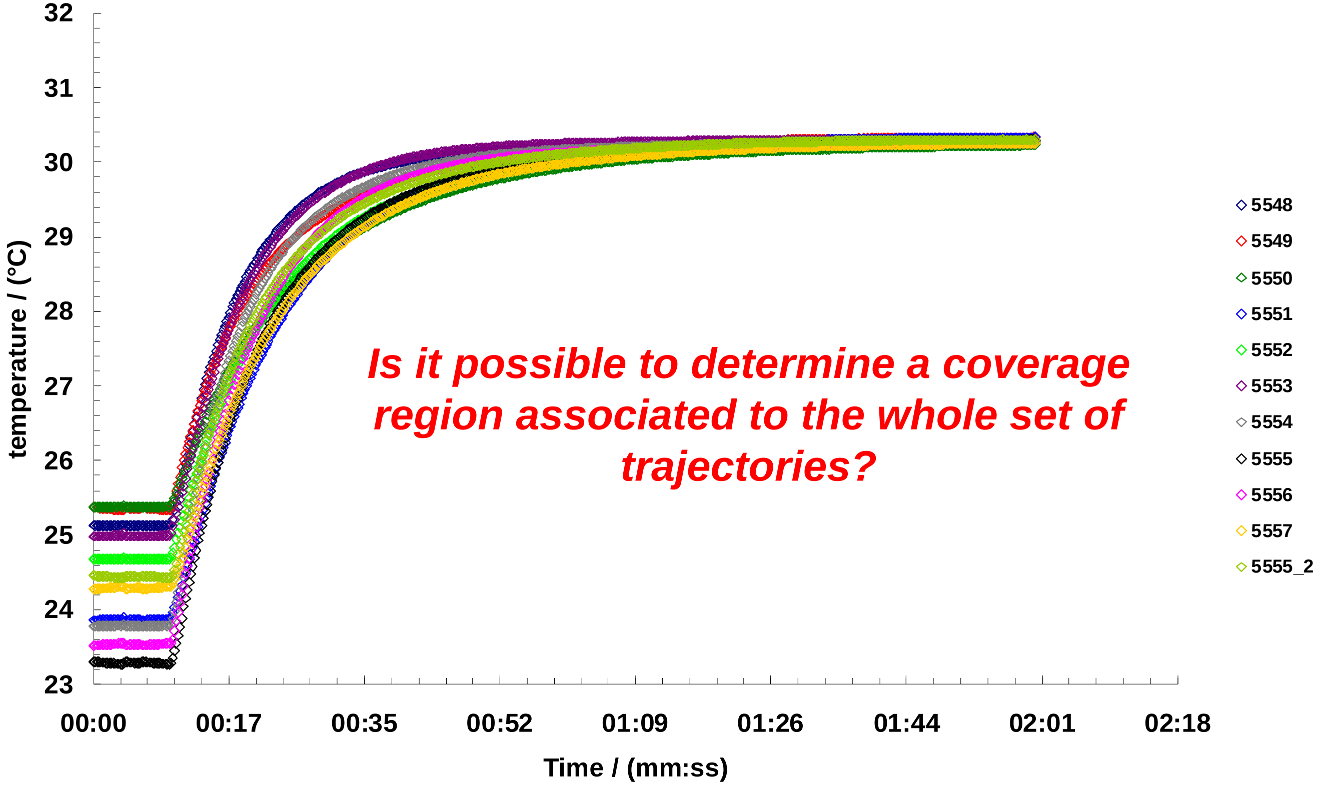


Sensor response to a step excitation

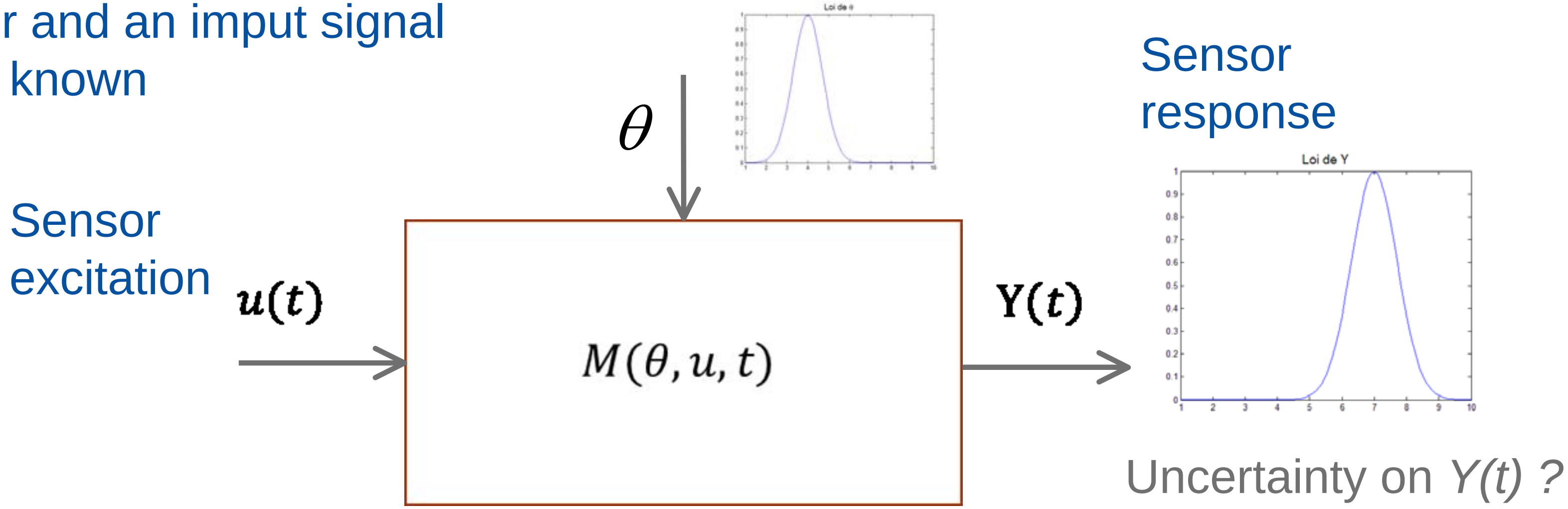
Unsteady state

- Real measurement are often unsteady
- Example: step excitation in temperature, **several probes**



Model based approach for uncertainty propagation

Relies on a model *M* of the sensor and an imput signal *u* well known



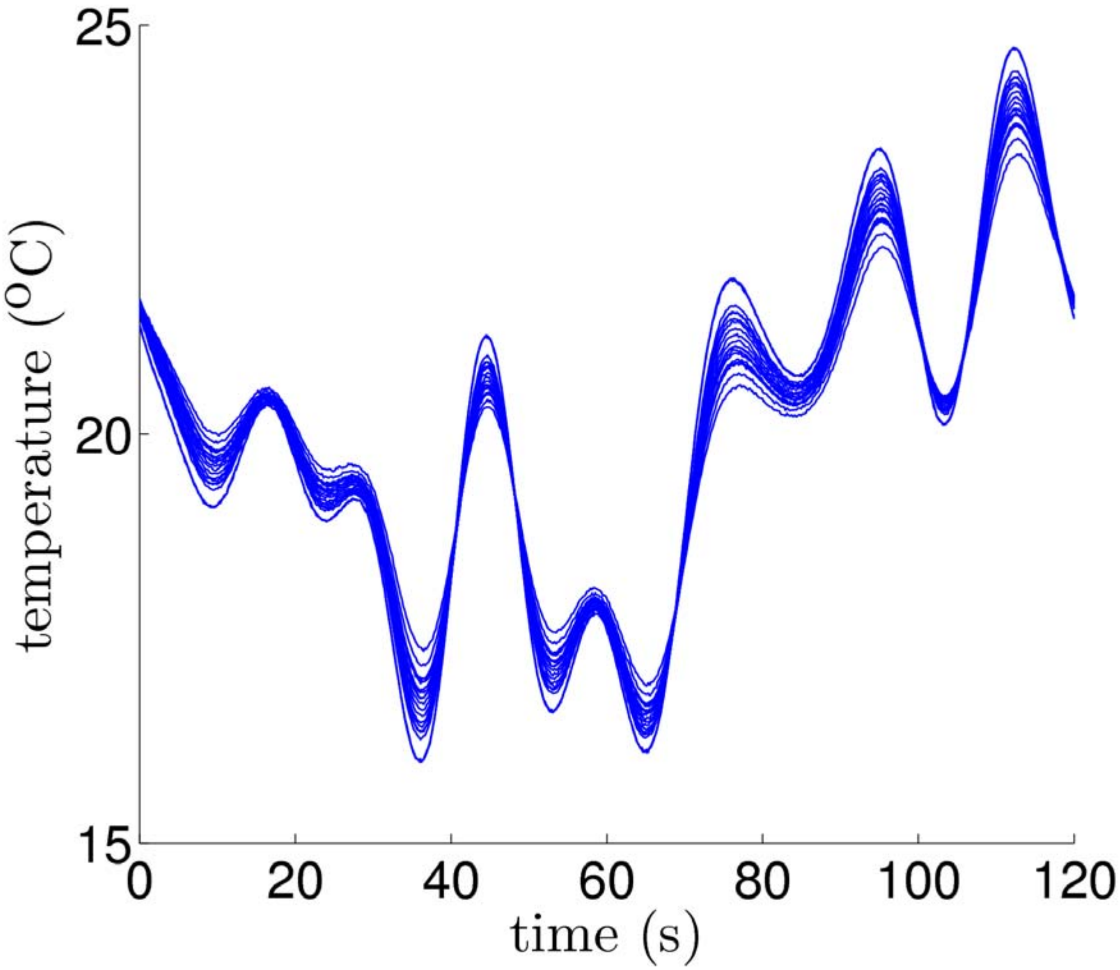
Particularly strong issues in industrial cases :

- It may be hard to find an accurate model of a sensor with a small number of parameters θ
- the excitation is often unknown or not well characterized

Measured signals for a set of 20 sensors

Simulation of observed data obtained by :

- arbitrary choice of the excitation signal *u(t)*
- considering sensors with different time constants



Modeling the sensor response

- The model of the response is unknown
- The observed signals suggest that the output trajectories are variations around a mean behaviour
- The number of tested sensors is small
- In the case of fluctuations varying slowly in time, it’s no longer valid to consider that the errors at two different times are independent

Statistical model and inference

The response $\boldsymbol{Y^{obs}}$ is modeled as a sum of :

• a Gaussian process $\boldsymbol{M}$ invariant through the sensors representing the nominal behaviour

• Gaussian processes $\boldsymbol{\mu}$ specific to each sensor $\boldsymbol{j}$, representing the departure from the nominal behaviour due to specific values of the conception parameters

• a random noise $\boldsymbol{\varepsilon}$ i.i.d. at each time $\boldsymbol{t_k}$, with the sensor $\boldsymbol{j}$ and the repetition $\boldsymbol{i}$

$$Y_{i,j,k}^{obs} = M(t_k) + \mu_j(t_k) + \varepsilon_{i,j,k}$$

$$\varepsilon_{i,j,k} \sim N(0, \sigma^2)$$

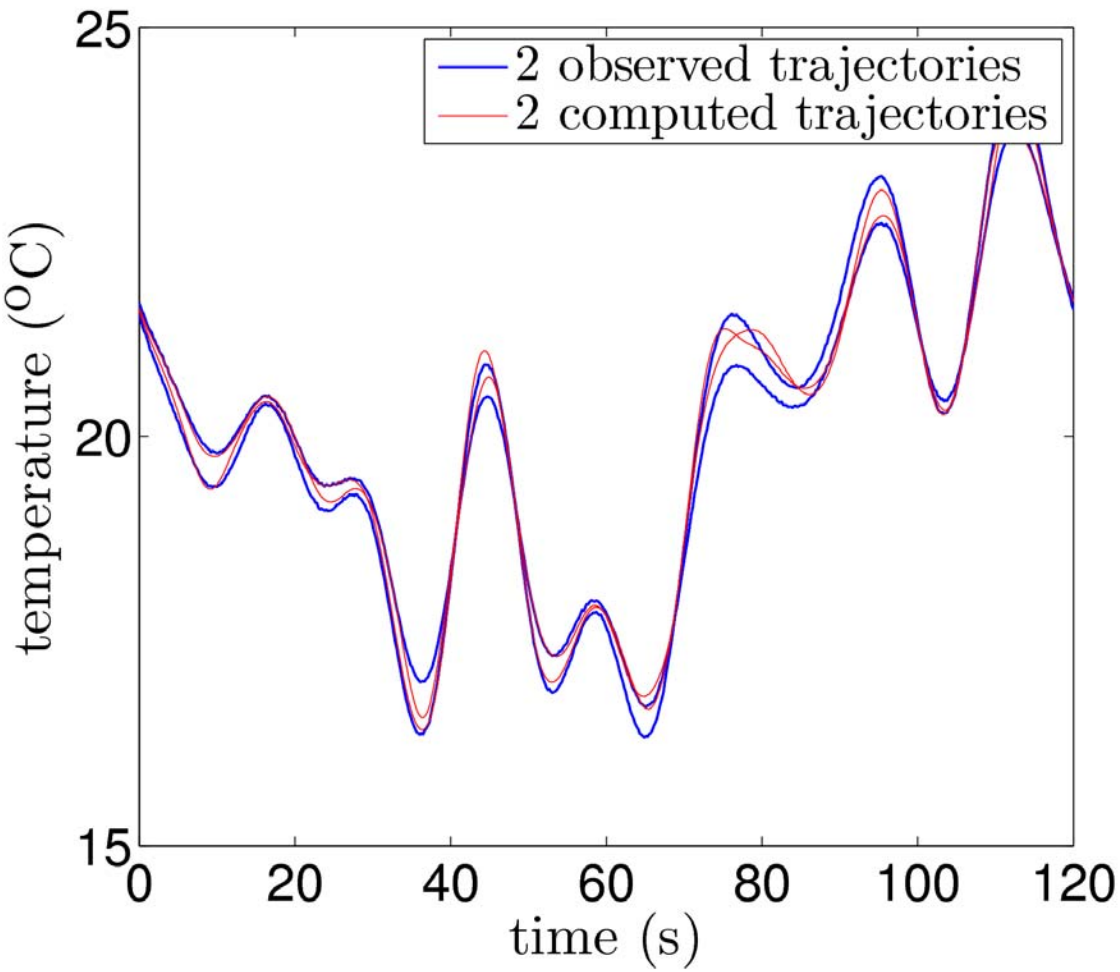
The model parameters are then estimated by likelihood maximisation

Observed trajectories and simulated trajectories

Considering the statistical model estimated, Monte Carlo simulations are performed to compute trajectories of the sensors

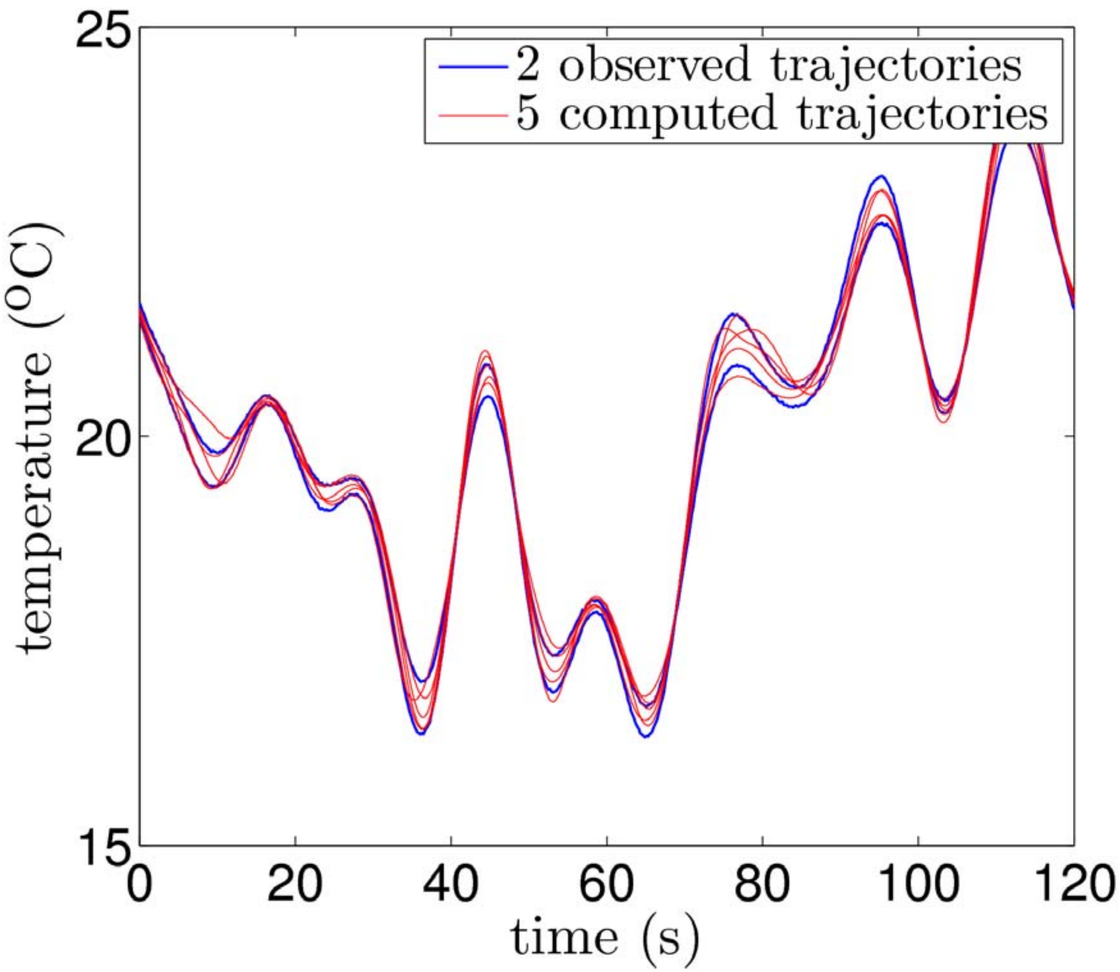
Blue plots: 2 trajectories picked from the 20 sensor outputs,

Red plots: 2 trajectories picked from the set of simulated trajectories



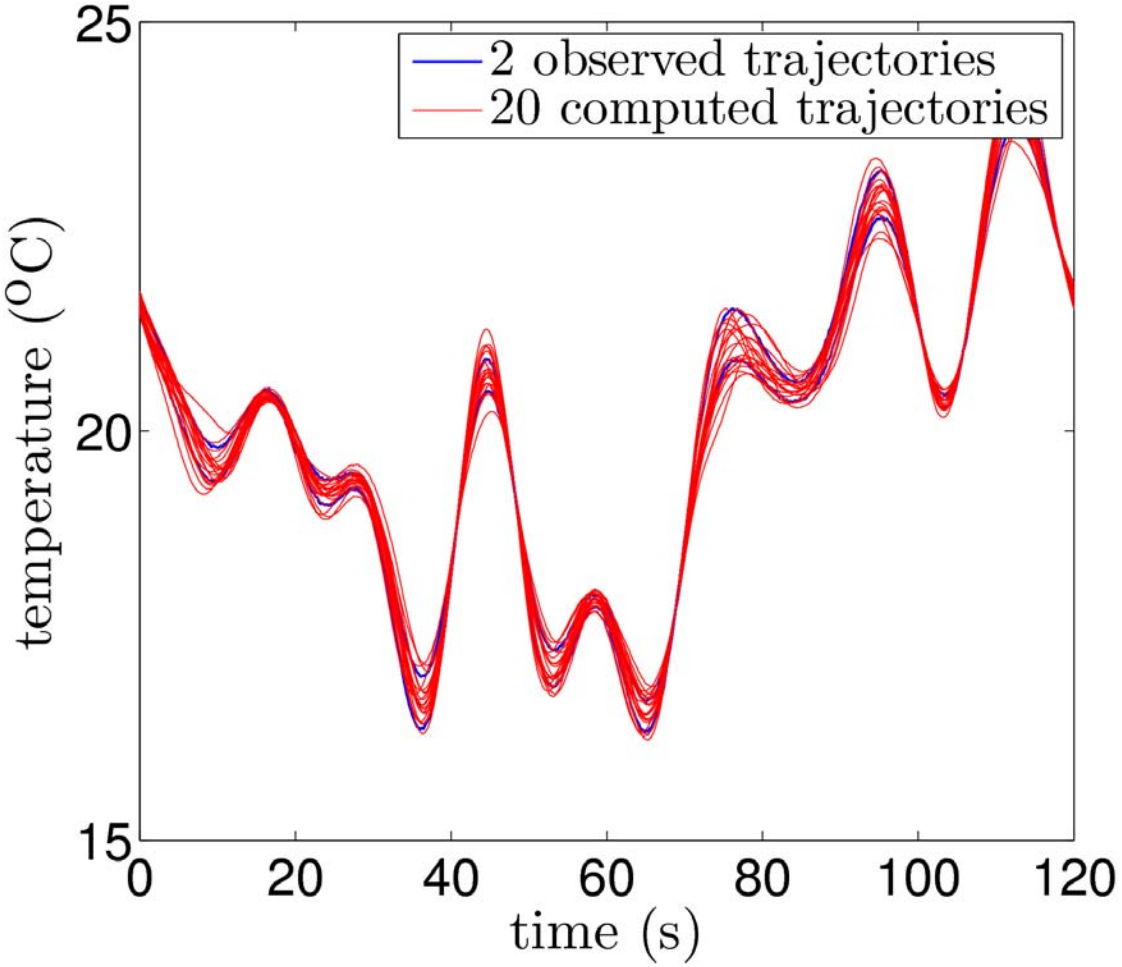
Observed trajectories and simulated trajectories

With 5 trajectories picked from the set of simulated trajectories



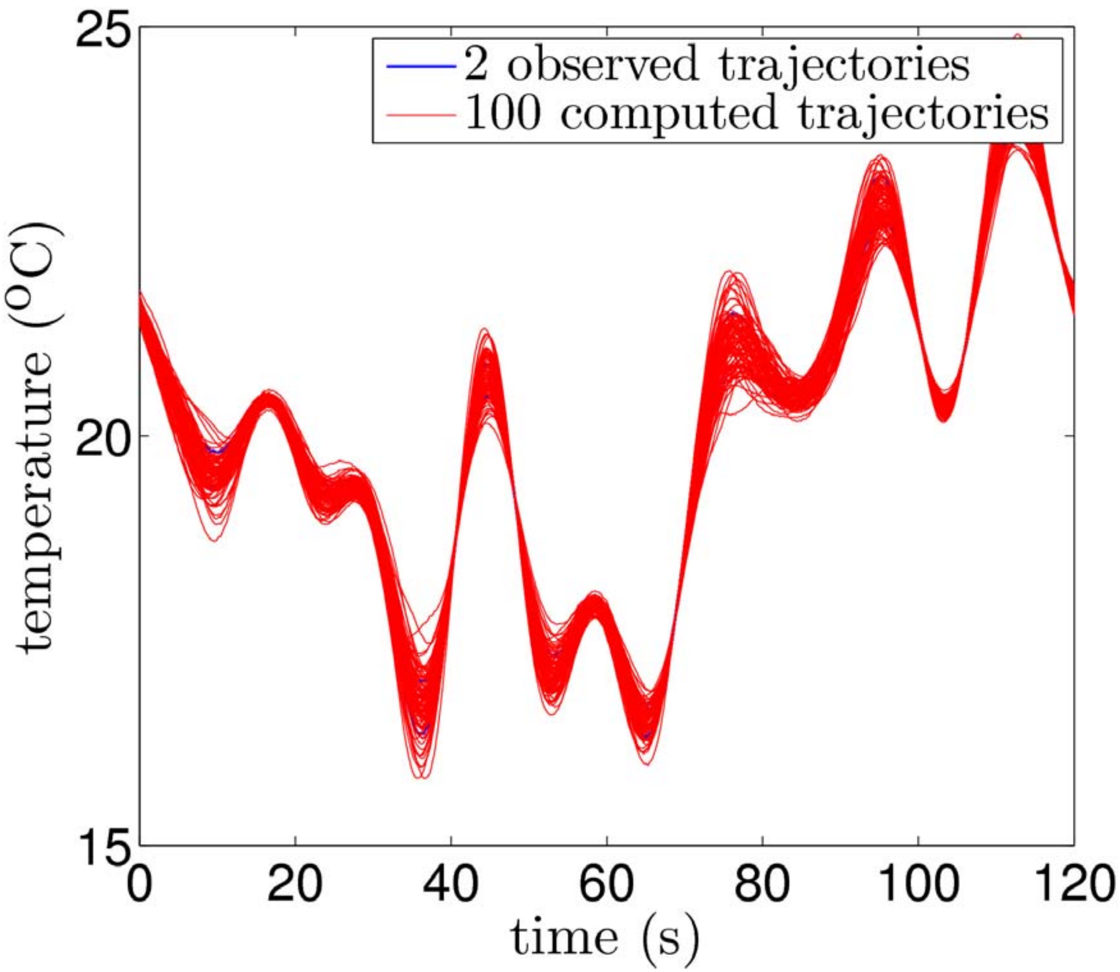
Observed trajectories and simulated trajectories

With 20 trajectories picked from the set of simulated trajectories



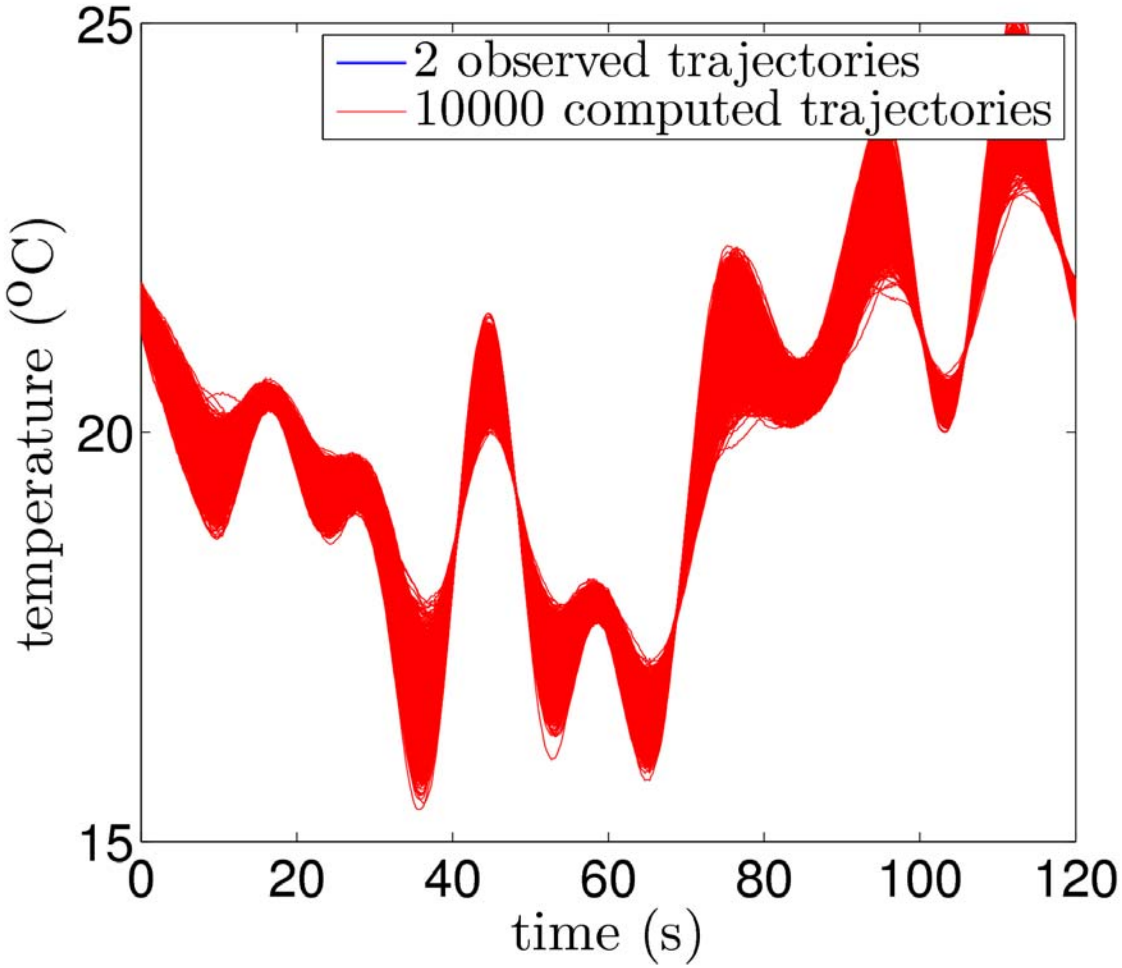
Observed trajectories and simulated trajectories

With 100 trajectories picked from the set of simulated trajectories



Observed trajectories and simulated trajectories

With the all set of simulated trajectories (10000 trajectories)



What is the uncertainty associated to the response of the sensors ?

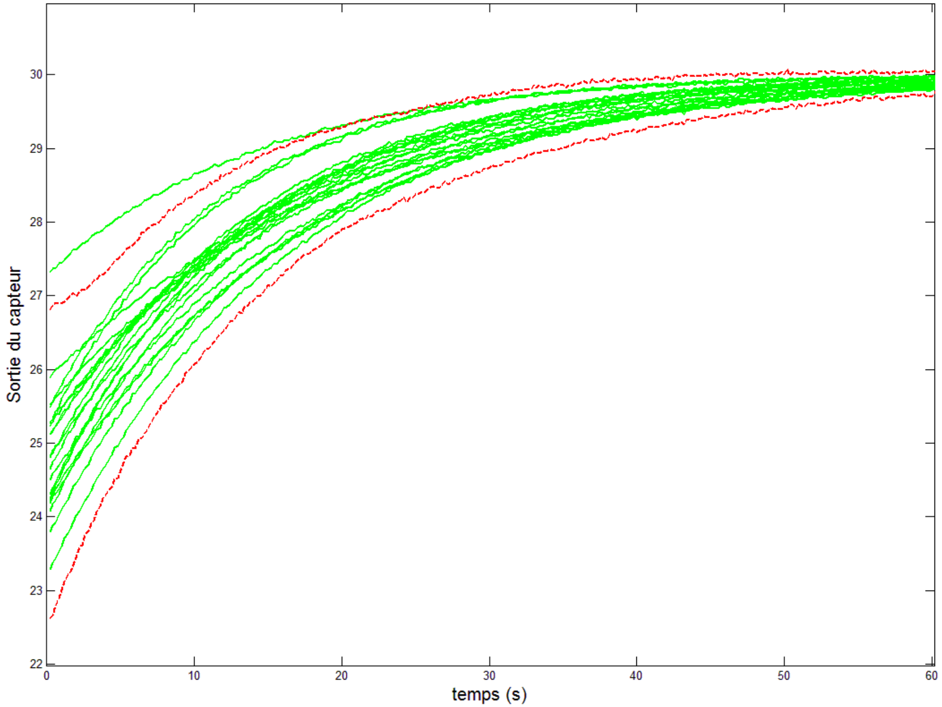
How to compute a coverage region associated to these trajectories ?

Coverage region

Defining a coverage region at a given level α

R_α is a coverage region, defined such that :

$$\mathbb{P}(\forall t \in \mathbb{T}, Y(t) \in R_\alpha) = \alpha$$



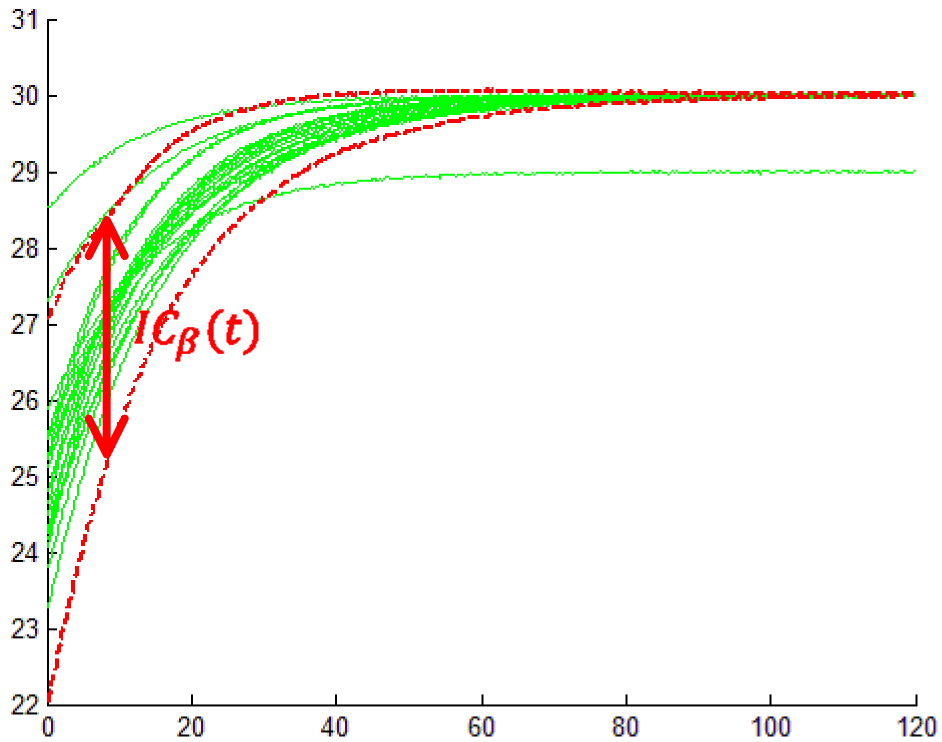
This region must contain α % of the trajectories

Proposal definition of the confidence region

Non unique way to define a coverage region

<u>Proposed process :</u>

1. To obtain confidence intervals CI_β at each instant t
2. To define coverage region of level α based on these intervals



Exemple :
$\boldsymbol{\beta = 95\% \rightarrow \alpha = 70\%}$
$\boldsymbol{\beta = 99\% \rightarrow \alpha = 90\%}$

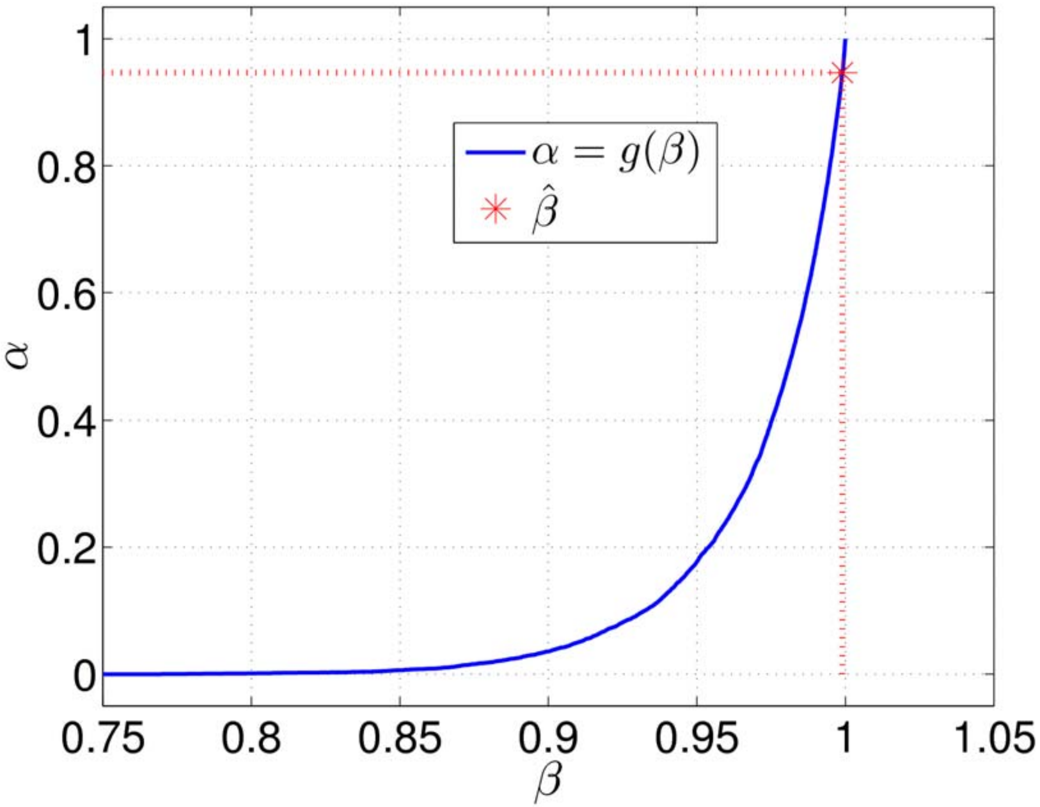
Mathematical definition of the coverage region R_α

R_α is a coverage region if, $t \in T, P(Y(t) \in R_\alpha) = \text{constant}$

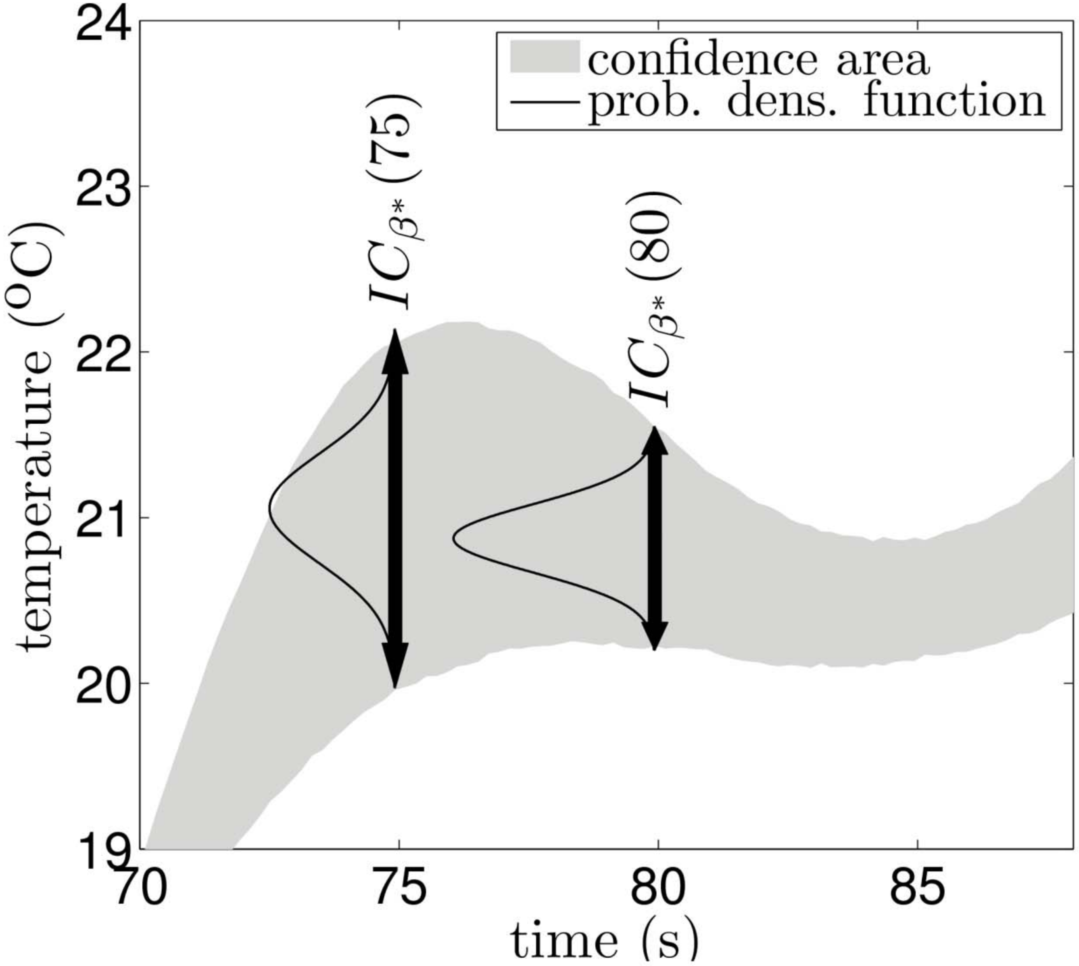
$R_{CI_\beta} = U_{t \in T} \{CI_\beta(t)\}$ defines a coverage region of level $\alpha = g(\beta) \leq \beta$

- Empirical estimation of the function *g(β)*
- Inversion of the function leads to find, for a desired level α, the value of β to be used

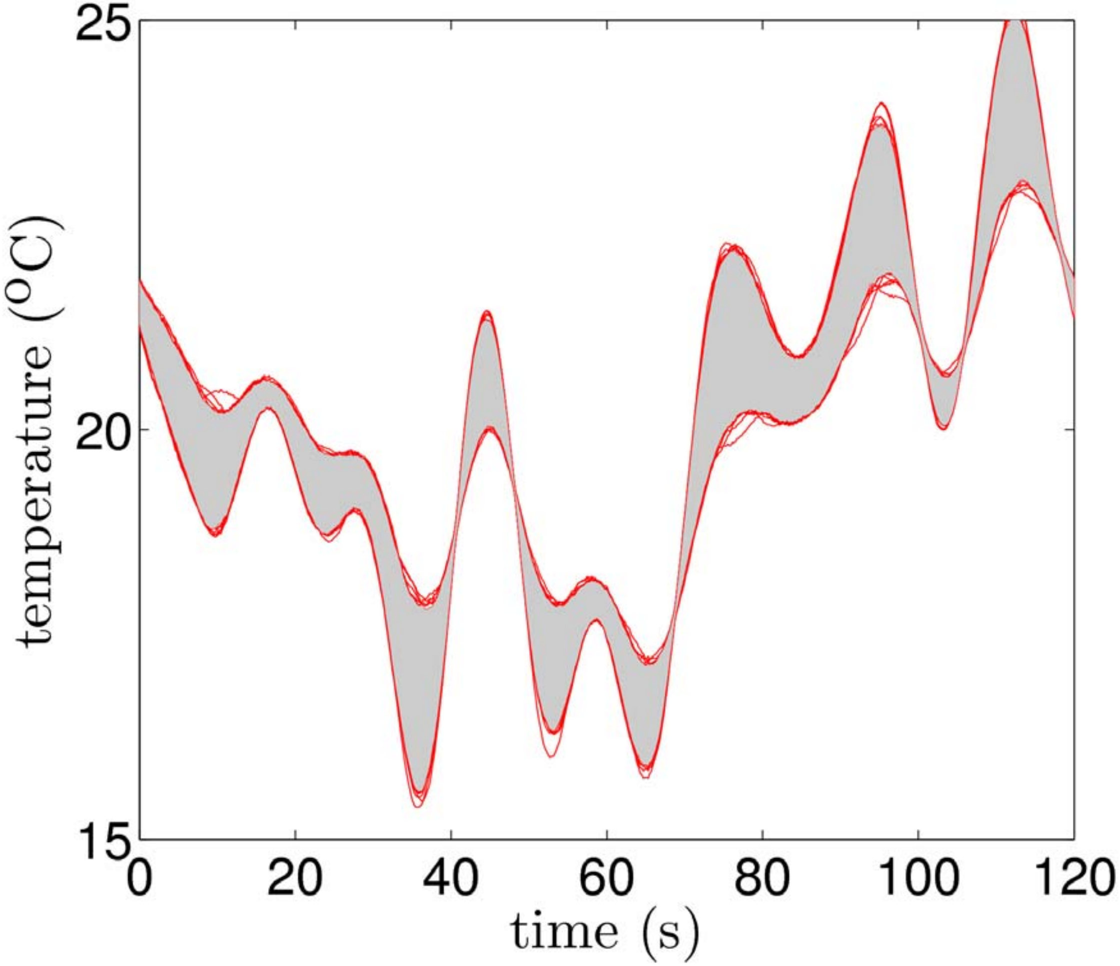
The red star indicates the value of β for α = 95%



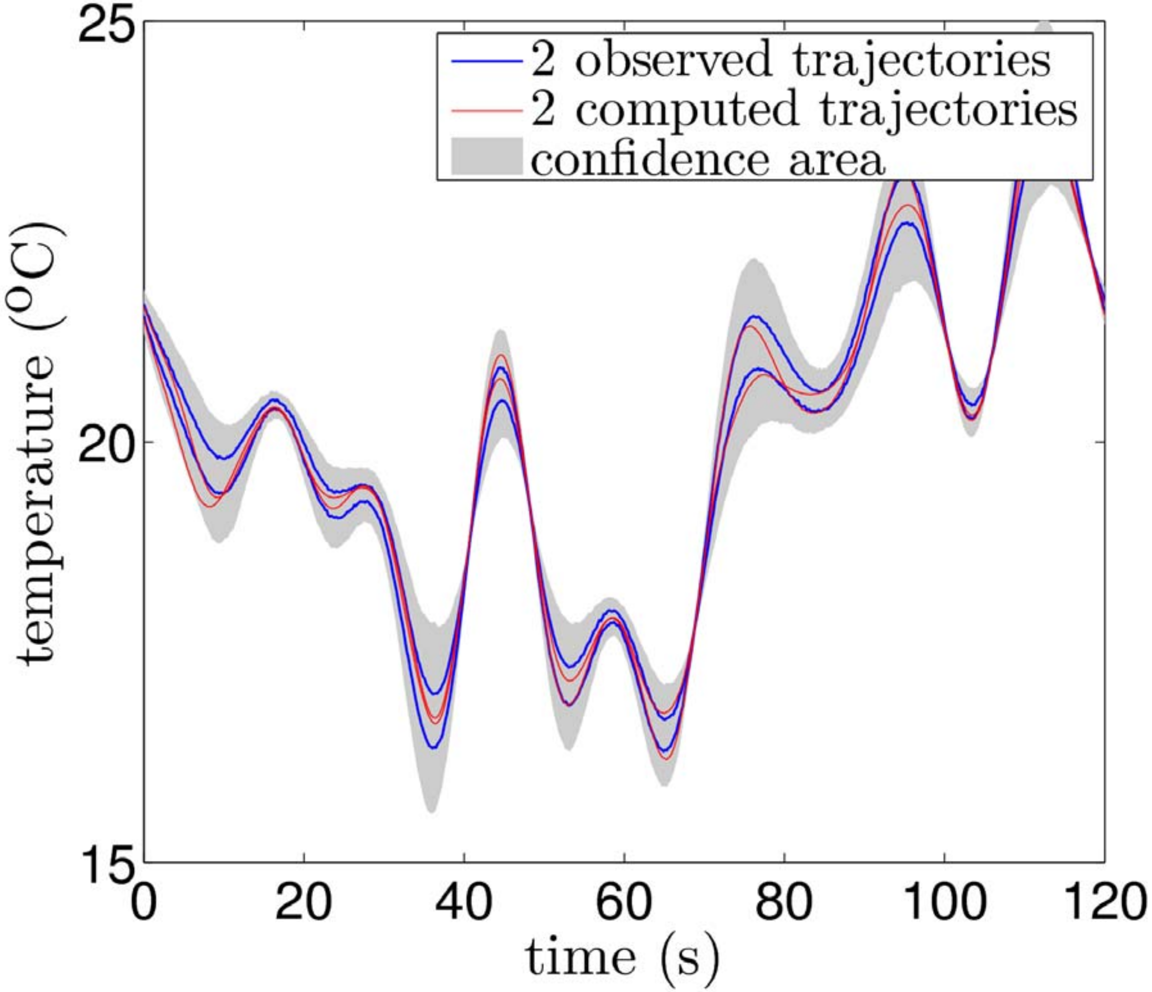
Construction of the coverage region R_α



Coverage region of level $\alpha = 95\%$



Adequacy between observed trajectories and R_α



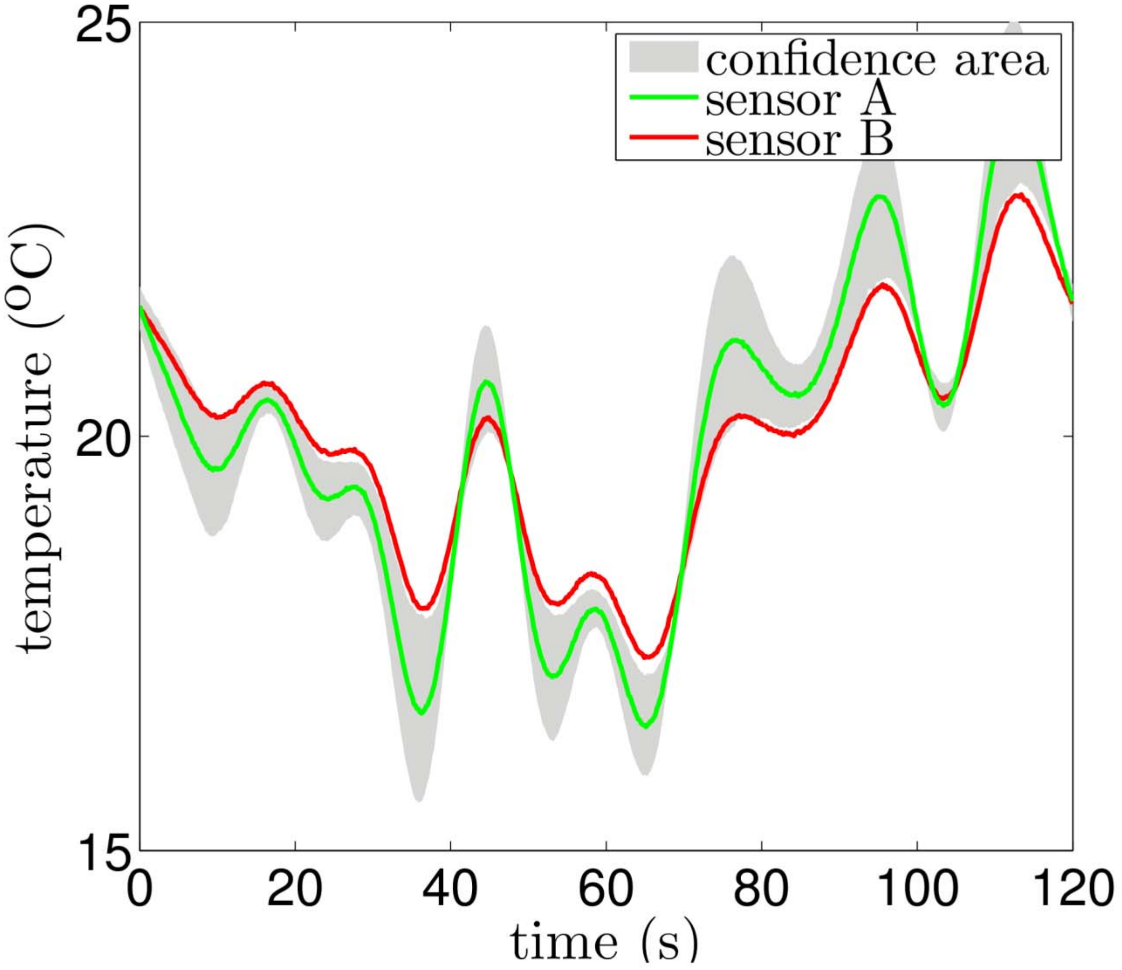
Quality control of the sensors

Among all simulations of sensors responses according to their own time constant, let's focus on two particular sensors A and B

The trajectory of sensor A lies within the coverage region previously estimated

The trajectory of sensor B lies outside the coverage region

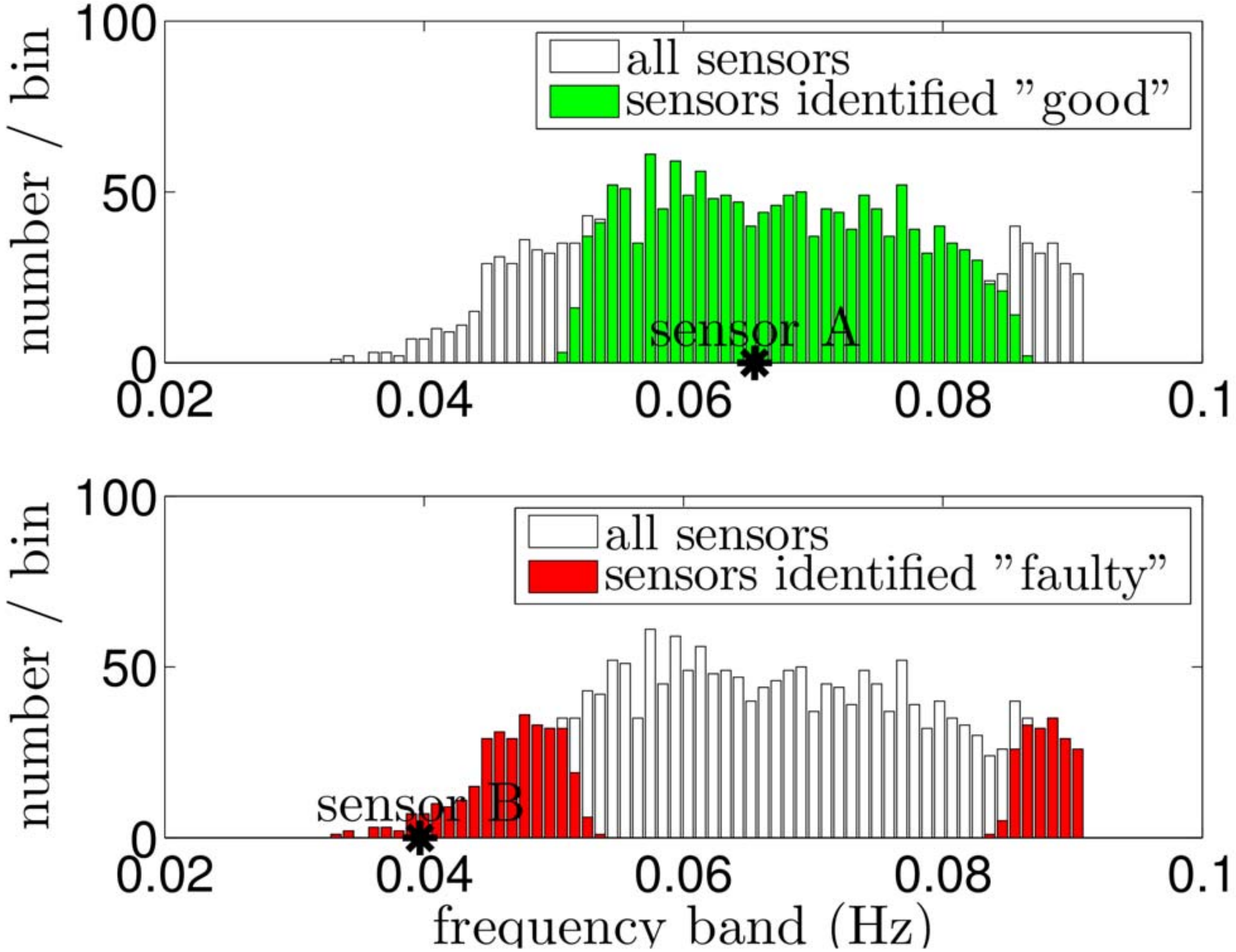
May we conclude that A is a "good" sensor and B a "bad" one ?



Identification of sensors *A* and *B*

The cut-off frequency is the inverse of the time constant of the sensor

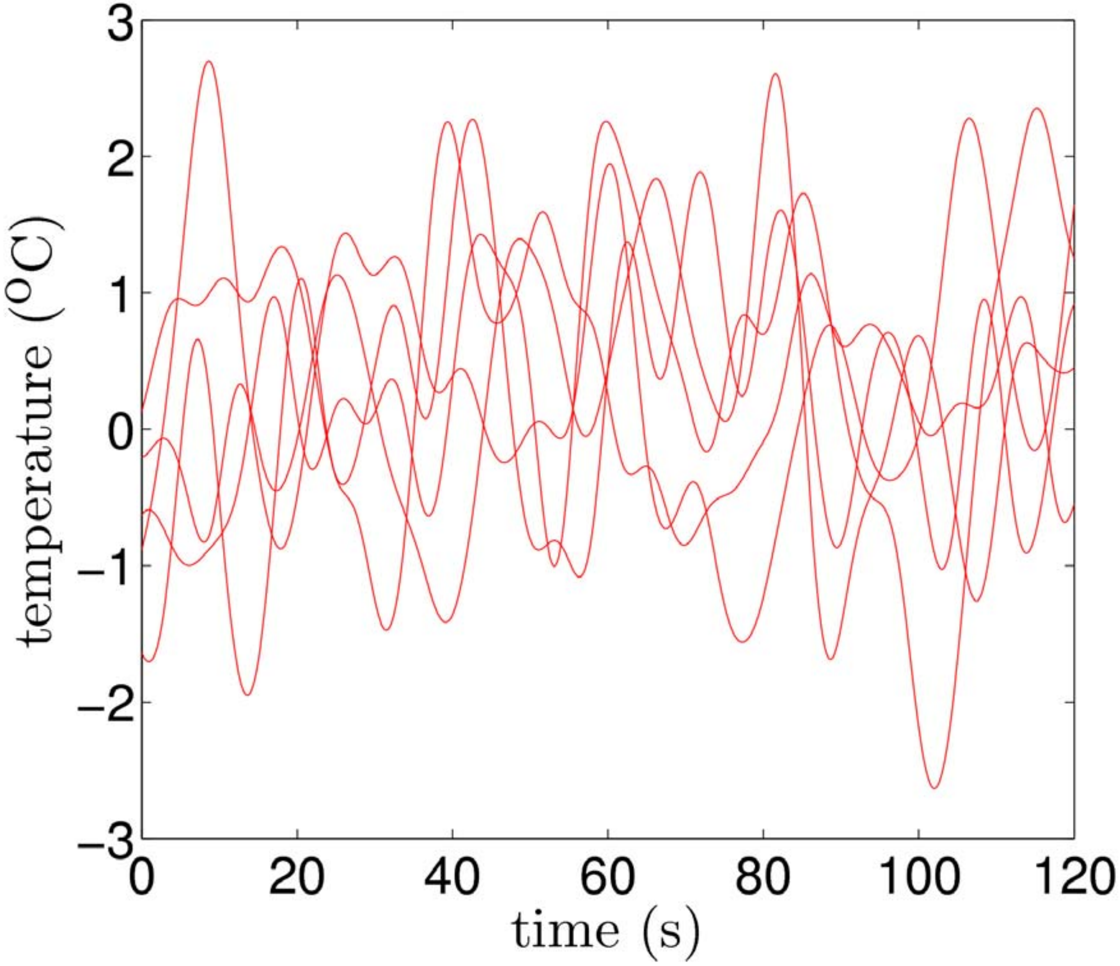
Sensor *A* has an expected frequency in the intermediate range such as all the sensors in green whose trajectories belong to the coverage region



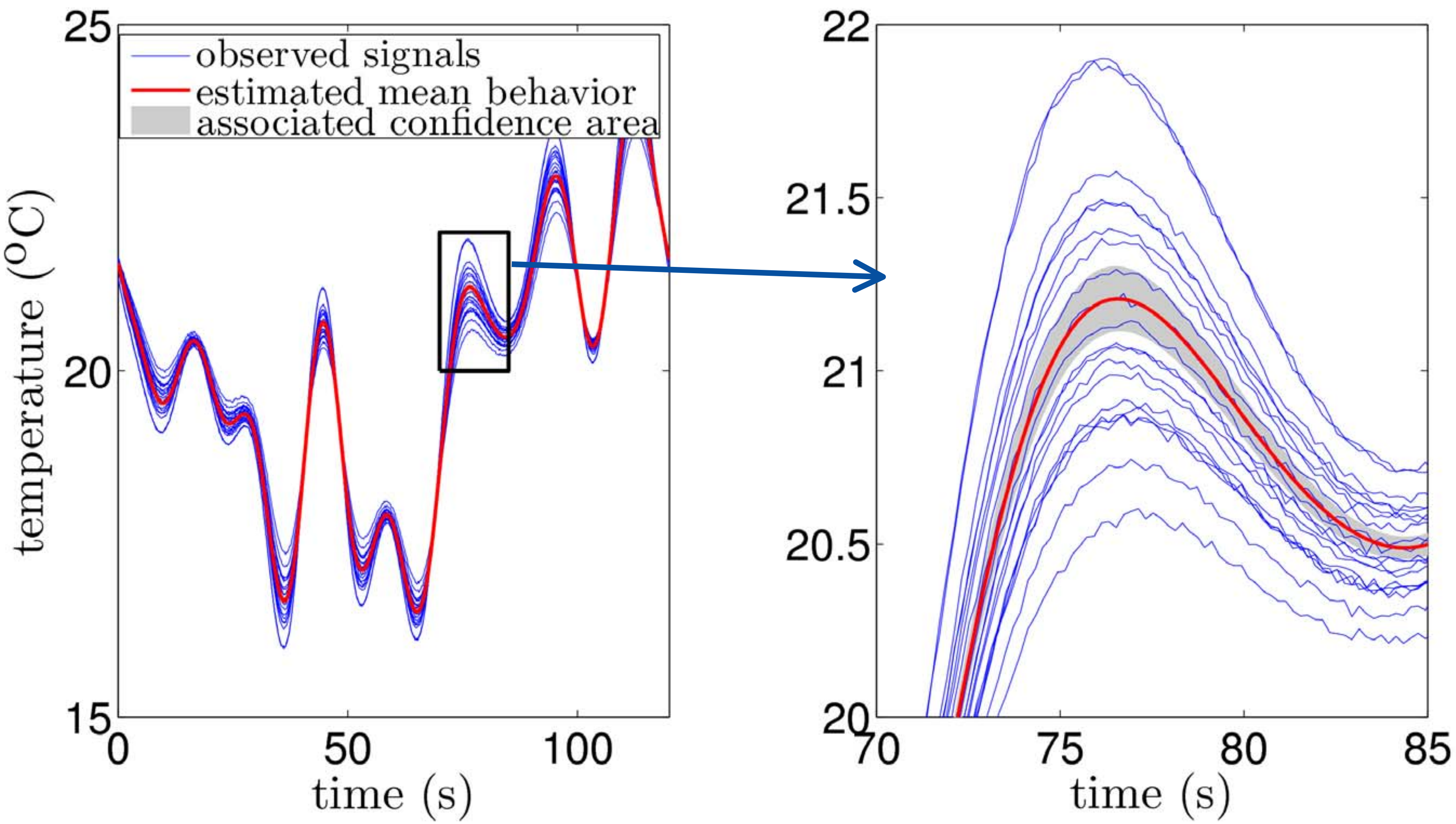
On the contrary, sensor B has a low frequency, indicating an inaccurate behaviour, like almost all the sensors outside the coverage region

Conclusion

- Proposal to characterize measurement uncertainty for dynamic measurement based on the concept of coverage region
- The estimation of the coverage region requires a statistical model of the sensor output
- The statistical model developed allows to catch both the mean behaviour and the magnitude of the departures from the mean behaviour
- A prior model of the sensor is not required
- Gaussian process is a suitable statistical tool to take into account the time dependence



3
2
1
0
−1
−2
−3
0
20
40
60
80
100
120
temperature (^{o}C)
time (s)


observed signals
estimated mean behavior
associated confidence area
temperature (°C)
25
20
15
0
50
100
time (s)
22
21.5
21
20.5
20
70
75
80
85
time (s)